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Highway Alignment Optimization through Feasible Gates

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An efficient optimization approach, called feasible gate (FG), is developed to enhance the computation efficiency and solution quality of the previously developed highway alignment optimization (HAO) model. This approach seeks to realistically represent various user preferences and environmentally sensitive areas and consider them along with geometric design constraints in the optimization process. This is done by avoiding the generation of infeasible solutions that violate various constraints and thus focusing the search on the feasible solutions. The proposed method is simple, but improves significantly the model's computation time and solution quality. Such improvements are demonstrated with two test examples from a real road project.

Keywords: Highway Alignment Optimization, User-defined Constraints, Feasible Bound, Horizontal Alignment, Vertical Alignment, Genetic Algorithms (GAs), Geographic Information Systems (GIS), Computation Efficiency, and Solution quality

I. Introduction

In a conventional highway design project, engineers and planners usually start by selecting several candidate alignments and then narrow their focus to the detailed alignment design. They consider various factors such as design specifications, costs, safety, and environmental impacts of candidate alignments in the planning stage. Among such factors, the effects of alignments on environmentally sensitive areas are

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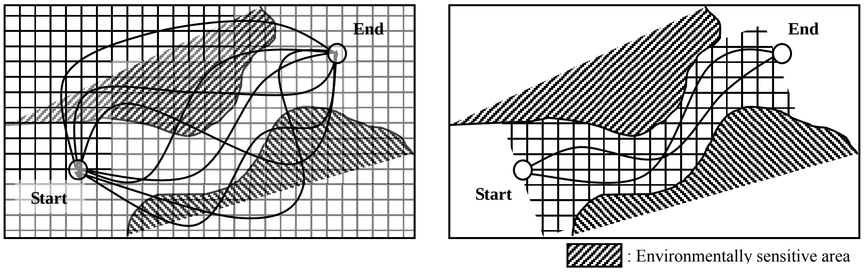
often the most important and complex effects. Political issues may also be critical in selecting rights-of-way. These factors are intangible and not easily estimated in monetary values; however, they may greatly reduce the alignment search problem by excluding many possibilities and requiring alignments to pass through some narrow "gates" or "corridors".

Our highway alignment optimization (HAO) model has been extensively refined in recent years to find the alignments that best satisfy various objectives and constraints during the initial stages of road projects (Jong, 1998; Jong and Schonfeld, 2000 and 2003; Jha, 2000; Jha and Schonfeld, 2000a and 2004; Jha et al., 2004; Kim, 2001). It integrates genetic algorithms (GAs) and a geographic information system (GIS) to solve the HAO problem and deals with various alignment-sensitive cost components associated with a road construction. A brief review of the HAO model is provided in the next section.

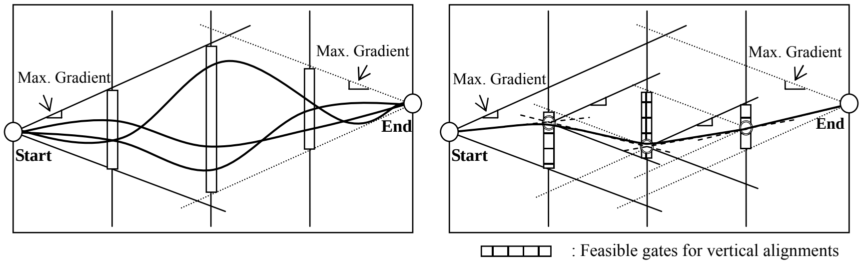
Recently, the model has been applied to an actual road construction project, the Brookeville Bypass, in which it has demonstrated its capabilities (Kang et al., 2005 and 2006). The model evaluates numerous possible alignments (horizontally and vertically), which may reflect various objectives and design standards. Moreover, it provides practical information to highway engineers and planners, such as various alignment-sensitive costs, station coordinates of horizontal and vertical alignments, and a mass diagram. However, in applying the model to a real road-construction project, it has been challenging to realistically represent various user preferences and environmentally sensitive areas and consider them in the optimization process. For instance, representation of user-specified narrow gates or specific intersections with existing roads that a new alignment should pass through has been difficult and required considerable model development.

Until recently the model has relied on a penalty approach to guide the search toward better solutions. It assigned penalties to the cost functions if the solution alignments violated the corresponding constraints and eventually screened out the candidate solutions whose constraint violations were significant. However, finding the feasible solutions that satisfy the user-defined constraints is computationally expensive since the model has to evaluate all generated solutions. As shown in Figure 1(a), many generated alignments may affect the existing environmentally sensitive areas since the search space is the entire area within the rectangular bounds. Such inefficiency is more severe if the sensitive areas are more complex so that the area of interest is also more complex or narrower. Obviously, the alignments that violate sensitive areas cannot be the best solutions; furthermore, the detailed evaluation of each solution takes considerable time. Thus, a good representation of feasible area of interest should be considered. An efficient use of the feasible search area reduces computation time as well as guarantees feasible solutions during the search process. The computational improvements are desirable because each candidate alignment requires massive processing of GIS data for its detailed evaluation.



(a) Baseline Horizontal Bound (**b)** Specified Horizontal Feasible Bound

Figure . 1. Bounded Horizontal Search Space



(a) Baseline Vertical Bounds **(b)** Specified Vertical Feasible Bounds

Figure 2. Bounded Vertical Search Space

In this paper we develop a feasible gate (FG) method (for horizontal (HFG) and vertical (VFG) alignments) to ensure that complex preferences and environmental requirements are satisfied efficiently in the search process of the HAO model. This approach is intended to avoid generating infeasible solutions that are outside the acceptable bounds and thus to focus the search on the feasible solutions. Figures 1(b) and 2(b) provide good insights into the proposed FG approaches for horizontal and vertical alignments, respectively. For both vertical and horizontal alignments the points of intersections (PI's) are only generated here (randomly, by genetic algorithms) along cutting planes orthogonal to the straight line connecting the start and end points, as shown in Figure 4 and 8. (More complex backtracking alignments are optimized Jong, 1998.) The key contribution in this work is to limit the fraction of the cutting planes within which PI's for alignments can be generated, both horizontally and vertically. These limited "gates" are based on user preferences and environmental factors for horizontal alignments and on allowable gradients for vertical alignments, after adjustments to allow PI's outside feasible regions if the curved alignments at those PI's stay within feasible regions. By avoiding the generation and evaluation of many infeasible alignments the search for optimized solutions is significantly accelerated.

Particularly for horizontal alignments, since various spatial considerations apply, the preferred horizontal feasible gates may be quite complex, discontinuous, and significantly depend on the preferences of model users. Therefore, ways of dealing with the various user preferences and reflecting them in the optimization process are key issues to be resolved. It is relatively easier to ensure feasible gates for vertical alignments than for horizontal ones. The feasible ranges are usually bounded by design standards, such as a maximum gradient.

Related Literature

Earlier studies anticipated that the application of mathematical models to highway design would significantly speed up the design process and result in better solutions (OECD, 1973; Shaw and Howard, 1982). Several categories of models have been developed for optimizing road alignments either horizontally or vertically or both other than our GAs and GIS based model. These are (i) calculus of variations (Howard et al., 1968; Shaw and Howard, 1981 and 1982), (ii) network optimization (Turner, 1971; Athanassoulis and Calogero, 1973; OECD, 1973; Parker, 1977; Trietsch, 1987; Turner, 1978), (iii) dynamic programming (Horgan, 1973; OECD, 1973; Puy Huarte, 1973; Murchland, 1973; Nicholson, 1976; Trietsch, 1987; Goh et al., 1988; Fwa, 1989), (iv) enumeration (Easa, 1988), (v) linear programming (ReVelle et al, 1997), and (vi) numerical search (Hayman, 1970; Pearman, 1973; Robinson, 1973). The advantages and disadvantages in each approach are briefly summarized in Table 1 and detailed discussions of those approaches may be found in Jong (1998) and in Jong and Schonfeld (2003). Some recent related literature is reviewed below.

de Smith (2006) provides a gradient and curvature constraint method to determine an optimal alignment for roads, railroads, and pipelines. He determines the optimal path based on four steps: (i) determination of initial shortest alignments that satisfy gradient constraints in a tilted planner surface, (ii) distance calculation of the alignments with elevation matrix, (iii) horizontal, and (iv) vertical path smoothing of the alignments with spline functions and curvature constraints. All four steps are provided with detailed procedures. However, since his method may require a conventional cost evaluation procedure for candidate alignments from the first four stages and those steps may not be automatically integrated, considerable time may be needed to obtain the final solutions. de Smith also discusses ways of dealing with obstacles or no-go areas of the alignments and involves them as additional constraints in step 1. However, these are very rough and all bounds are parallel to the straight line between the start and endpoints of the alignment; thus, they cannot realistically represent real shapes of the untouchable areas on a nonplanar surface, as in a realistic GIS. Fwa et al. (2002) optimize vertical alignments with the assumption that horizontal alignments are initially given. They consider various design constraints on vertical alignments, such as gradient, curvature, fixed point, critical length of grade, and non-overlapping of horizontal and vertical curves. They also use genetic algorithms to optimize the vertical alignments. A constant static penalty function is applied to avoid infeasible solutions in the optimization process. However, it should be noted that such a static constant penalty may cause serious errors since it introduces large unsmooth steps into the optimization search process.

Much progress has been made in developing models for optimizing vertical alignments over the past three decades (Jong and Schonfeld, 2003); most studies above only attempt to optimize vertical alignments. The progress in developing models for optimizing horizontal alignments or 3-dimensional alignments (i.e. jointly optimizing the horizontal and vertical alignments) is very limited and the number of such models is small. The main reason is that modeling horizontal alignment is quite complex and requires substantial data for various cost components, such as right-of-way cost (i.e., land-acquisition cost) and pavement cost, and other political or environmental issues.

In the HAO model, the horizontal and vertical alignments are generated simultaneously. The alignments are first specified by series of points of intersections (PI's) with XYZ coordinates and optimized jointly in 3-dimensional space. Then, these points are used to create the horizontal and vertical alignments simultaneously with the curve fitting algorithms embedded in the model. Finally, the relevant cost components affected by the 3-dimensional alignments are comprehensively evaluated. It should be noted that simultaneous 3-D optimization is clearly preferable to sub-optimizing the vertical alignment for a previously sub-optimized horizontal alignment since such a conditional optimizing process (sub-optimizing a horizontal alignment first and then sub-optimizing the vertical alignment based on the horizontal alignment created) is less likely to avoid local optima. The model assumptions, search procedure, and cost components embed in the model are briefly described in the next section. This should help readers understand the context of the proposed FG method, which is designed for improving search efficiency of the HAO model by avoiding the generation of infeasible solutions and detailed their time-consuming evaluation. The following sections introduce the proposed algorithm and demonstrate its performance with an example study. We conclude with comments on the effects and limits of the proposed FG method together with extensions of further study for improvements in the highway alignment optimization process.

Table 1. Advantages and Disadvantages of Existing Approaches for Optimizing Road Alignments

Method	Advantages	Disadvantages
Calculus of Variations	- Yields smooth alignment - Possibly finds the global optimum - Has continuous search space	- Cannot deal with discontinuous cost items (requires well-developed objective function) - Complex modeling and computation efforts
Network Optimization	- Is simple and easy to use - Can use well-developed algorithms for solving the problem exist - Possibly finds the global optimum	- Cannot yield smooth alignment - Uses discrete solution set rather than continuous search space - Requires large memory
Dynamic Programming	- Simple and easy to use - Can use well-developed algorithms for solving the problem exist - Possibly finds the global optimum	- Can not yield smooth alignment - Uses discrete solution set rather than continuous search space - Requires large memory - Has difficulty in handling backward bends

Enumeration	- Can yield a realistic alignment - Possibly finds the global optimum - Can consider most of the important constraints	- Is inefficient - Uses discrete solution set rather than continuous search space
Linear Programming	- Is simple and easy to use - Can use well-developed algorithms - Possibly finds the global optimum - Can yield smooth alignment - Has continuous search space	- Uses formulation only for limited cost items and constraints (must be linear) - Gradient and curvature constraint are formulated for a limited number of points
Numerical Search	- Can yield a realistic alignment - Can consider most of the important constraints and various costs - Has continuous search space	- Produces multiple local optima - Complex modeling and computation efforts - Has difficulty in modeling discontinuous cost items
GA-GIS Based Optimization (HAO model)	- Can yield a realistic alignment (horizontally and vertically) simultaneously - Possibly finds the global optimum - Has continuous search space - Is simple and easy to use - Can use well-developed algorithms for solving the problem exist - Can consider most of the important constraints - Can handle alignments with backward bends	- Depends on the accuracy of the spatial information (GIS databases) - May require high level computer environments (e.g., memory and hard drive capacity) if the study area is enlarged

* Source: Jong (1998)

II. Overview of the HAO Model

In the highway alignment optimization (HAO) model, a solution algorithm evolved from genetic algorithms (GAs) is used for optimization process and a geographic information system (GIS) is used to realistically reflect a real-world problem. Several assumptions in the integrated GAs and GIS based HAO model are listed below.

i. The start and end points of the alignment are given.
ii. The points of intersection (PI's) for horizontal and vertical alignments are designed to lie on the cutting planes orthogonal to the straight line connecting the start and end points (refer to Figures 4 and 8).
iii. Cost functions incorporated in the model can consider all costs that depend significantly on the alignment.
iv. Design constraints considered in the model are based on AASHTO standards.
v. The input GIS databases are sufficiently accurate for the evaluation of alternative alignments.

In our HAO model, it is currently assumed that the start and end points of the alignment are predetermined by model users before the optimization. This assumption reduces the search requirements but should be relaxed in future model developments.

The model employs the GA-based algorithm to search for optimized alignments connecting the given endpoints while evaluating fitness of various possible alternatives and exploiting the GIS database for assessing the environmental impacts and right-of-way cost. In the model, the basic alignment optimization problem is reduced to finding the points of intersection (PI's) for horizontal and vertical alignments (Jong, 1998; Jong and Schonfeld, 2003). The PI's, which are represented as genes in the model, are created with genetic operators and designed to lie on the orthogonal-cutting planes so that each PI has unique x, y, and z coordinates. To create a horizontal alignment, the PI's are first connected with straight lines ("tangents") and curves are then fitted to connect the tangents. The corresponding vertical alignment is also determined by fitting parabolic curves at every PI. After creating the 3-dimensional alignments (horizontal and vertical alignments) simultaneously, the model starts to evaluate the alignments. Station coordinates of the alignment are transmitted to the GIS module in order to obtain the environmental impact summary and the right-of-way cost, while the other cost components are calculated in the optimization module. Then, the model evaluates the alignment with its total cost function. It is noted that the total cost function is the model's objective function having two main alignment-sensitive costs (supplier cost and user cost) and the alignment's environmental impacts calculated from the GIS module are used as constraints. The supplier cost consists of right-of-way cost, construction and pavement cost, earthwork cost, and bridge cost (including those crossing streams, valleys and existing roads). The user cost includes the cost of vehicle operation, travel time, and accidents. The right-of-way cost is calculated from the cost of the area taken by the alignment and damage to affected properties (Jha and Schonfeld, 2004), the construction and pavement cost is estimated from the road length multiplied by a unit cost, and the earthwork cost is calculated based on the actual ground elevations in the study area. Besides the alignment-sensitive costs, several penalty cost components are also included in the total cost function to ensure compliance with the AAHSTO design standards (such as, minimum horizontal curve radius, minimum superelevation runoff length, minimum length of vertical curves, maximum allowable gradient, critical length of grade length, and sight distance). Detailed model formulations and algorithms are provided in earlier publications (Jong, 1998; Jong and Schonfeld, 2000 and 2003; Jha, 2000; Jha and Schonfeld, 2000a and 2004) and not repeated in this paper.

III. Feasible Gate Approaches in Highway Alignment Optimization

3.1. Horizontal Feasible Gates (HFG) for Optimal Search

A horizontal feasible gate (HFG) approach is developed to represent a complex horizontal search space realistically to the HAO model process. In addition, since it requires interactive use of the spatial information in the study area, an input data preparation module (IDPM) is also developed. With incorporation of the IDPM into the HFG-based HAO model, we now enable users to interactively specify their preferences (e.g., areas of interest) on given GIS maps and enhance the model solution quality and computation efficiency

3.1.1. Input Data Preparation Module (IDPM) for User-Defined Horizontal Feasible Bound

The IDPM is a customized GIS with ArcView GIS 3.x. Figure 3 shows how the existing GIS maps and user's areas of interest are converted to the model-readable format through the IDPM. It is noted that digitized land use and property information (e.g., values and boundaries) maps are essential in using IDPM. Since GIS databases are widely used, some (even property maps) are available nowadays free or with some charges at the USGA, ESRI, and other websites of companies and local governments.

Let *BSA* be a baseline study area in which properties are spatially distributed in a rectangular space and *k* be a clipped property piece resulting from the superimposition of different map layers (refer to Figure 3). Additionally, let *U* be our area of interest, *U′* be the area outside it and *E* and *E′* be environmentally sensitive and insensitive areas, respectively. Then U_k denotes whether *k* is inside *U* and E_k indicates whether it is inside *E*. These variables are used to represent horizontal feasible bounds (*HFB*), untouchable areas (*HFB′*), and the set of feasible gates for PI's in the next section.

$$U_k = \begin{cases} 0 : \text{If property piece } k \text{ is outside } U \\ 1 : \text{If property piece } k \text{ is inside } U \end{cases} \qquad E_k = \begin{cases} 0 : \text{If property piece } k \text{ is outside } E \\ 1 : \text{If property piece } k \text{ is inside } E \end{cases}$$

$$k \in \begin{cases} HFB & \text{if } U_k\text{=1 and } E_k\text{=0, } \forall k \text{ in } BSA \\ HFB' & Otherwise \end{cases}$$

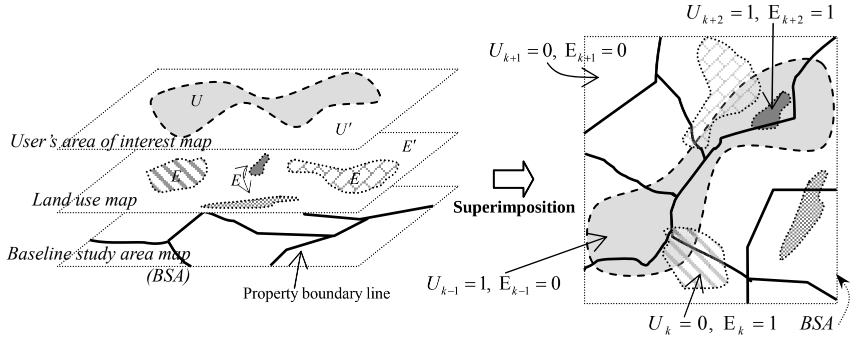


Figure 3. Setup of User-Defined Horizontal Feasible Bound with IDPM

3.1.2. Representation of Horizontal Feasible Gates with the Defined Feasible Bound

Let $Start_H = (x_s, y_s)$ and $End_H = (x_e, y_e)$ be horizontal start and end points of a new alignment, and $\overline{SE}$ denotes the line connecting $Start_H$ and End_H. Jong (1998) introduced vertical cutting lines, which are perpendicular to $\overline{SE}$, to find horizontal PI's of the alignment along the cutting lines (perpendicular to the straight line ($\overline{SE}$) connecting the two end points, as shown in Figure 4) in a rectangular search space. We adopt that concept in this paper to realistically represent the set of horizontal feasible gates (HFG) with the specified-horizontal feasible bounds (*HFB*) as shown in Figure 4. Jong's key variables and equations required to express our proposed method are as follows.

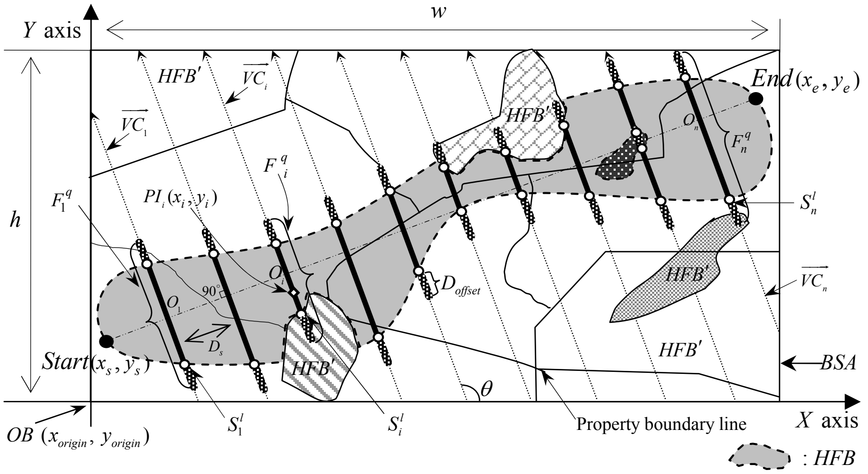


Figure 4. Representation of Feasible Gates for PI's on a Given Map Coordinate System

Suppose that we cut $\overline{SE}$ n times at equal distances between contiguous cuts and let $O_i = (x_{oi}, y_{oi})$ be the origin of the set of vertical cutting lines, $\overline{VC_i}$ $\forall i = 1, \ldots, n$. Then the coordinates of O_i are expressed as:

$$\begin{bmatrix} x_{o_i} \\ y_{o_i} \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \end{bmatrix} + \frac{i}{n+1} \begin{bmatrix} x_e - x_s \\ y_e - y_s \end{bmatrix} \tag{1}$$

Let θ be the angle between the cutting line and the X axis of the given map coordinate system. Then θ can be determined by the following equation:

$$\theta = \tan^{-1}((y_n - y_0)/(x_n - x_0)) + 90^\circ \tag{2}$$

where $0 < \theta < \pi$. We now let $OB = (x_{origin}, y_{origin})$ be the origin of the baseline study area (*BSA)* and h and w be height and width of the study area, respectively. Then the i_{th} vertical cutting line vector, $\boldsymbol{VC_i}$ can be defined as the function of θ, O_i, OB, h, and w. Additionally, let d_i be the coordinate of the intersection point at the i_{th} vertical cutting line and d_{iU} and d_{iL} be its upper and lower bounds. Then $\boldsymbol{VC_i}$, d_{iU}, and d_{iL} are determined based on the following four cases:

Case 1: If $\theta = 0$ or π

$$\begin{aligned} d_{iU} &= \left(x_{origin} + w\right) - x_{o_i} \\ d_{iL} &= \left(x_{origin} - x_{o_i}\right) \\ \overrightarrow{VC_i} &= \overrightarrow{(x_{origin},\, y_{o_i})(x_{origin} + w,\, y_{o_i})} \end{aligned} \tag{3a}$$

Case 2: If $0 < \theta < \pi / 2$

$$\begin{aligned} d_{iU} &= \min\left\{ {}^{\left((x_{origin} + w) - x_{o_i}\right)}\!\Big/_{\cos\theta},\; {}^{\left((y_{origin} + h) - y_{o_i}\right)}\!\Big/_{\sin\theta} \right\} \\ d_{iL} &= \max\left\{ {}^{\left(x_{origin} - x_{o_i}\right)}\!\Big/_{\cos\theta},\; {}^{\left(y_{origin} - y_{o_i}\right)}\!\Big/_{\sin\theta} \right\} \\ \overrightarrow{VC_i} &= \overrightarrow{(x_{o_i} + d_{iL}\cos\theta,\, y_{o_i} + d_{iL}\sin\theta)(x_{o_i} + d_{iU}\cos\theta,\, y_{o_i} + d_{iL}\sin\theta)} \end{aligned} \tag{3b}$$

Case 3: If $\theta = \pi / 2$

$$\begin{aligned} d_{iU} &= \left(y_{origin} + h\right) - y_{o_i} \\ d_{iL} &= \left(y_{origin} - y_{o_i}\right) \\ \overrightarrow{VC_i} &= \overrightarrow{(x_{o_i},\, y_{origin})(x_{o_i},\, y_{origin} + h)} \end{aligned} \tag{3c}$$

Case 4: If $\pi / 2 < \theta < \pi$

$$\begin{aligned} d_{iU} &= \min\left\{ {}^{\left(x_{origin} - x_{o_i}\right)}\!\Big/_{\cos\theta},\; {}^{\left((y_{origin} + h) - y_{o_i}\right)}\!\Big/_{\sin\theta} \right\} \\ d_{iL} &= \max\left\{ {}^{\left((x_{origin} + w) - x_{o_i}\right)}\!\Big/_{\cos\theta},\; {}^{\left(y_{origin} - y_{o_i}\right)}\!\Big/_{\sin\theta} \right\} \\ \overrightarrow{VC_i} &= \overrightarrow{(x_{o_i} + d_{iL}\cos\theta,\, y_{o_i} + d_{iL}\sin\theta)(x_{o_i} + d_{iU}\cos\theta,\, y_{o_i} + d_{iL}\sin\theta)} \end{aligned} \tag{3d}$$

Detailed explanations of the above equations are provided in Jong (1998) and Jong and Schonfeld (2000).

Let PI_i be the horizontal point of intersection corresponding to i_{th} vertical cutting line vector ($\boldsymbol{VC}_i$) and $S_i^{\,l}$ be the l_{th} intersection point of $\boldsymbol{VC}_i$ with property pieces that are in the specified horizontal feasible bounds (*HFB*) where l = 1,…, m_i and m_i is the total number of intersection points of $\boldsymbol{VC}_i$ with the property pieces in the *HFB*. Then the q_{th} horizontal feasible gate for PI_i, denoted as $F_i^{\,q}$ can be determined by a line segment connecting the two consecutive intersection points ($S_i^{\,l}$ and $S_i^{\,l+1}$) and an additional allowable offset (denoted by D_{offset}) where q = 1,…, $m_i/2$. As shown in Figure 4, the set of horizontal feasible gates $F_i^{\,q}$, $\forall i$ and $\forall q$, outlines the specified horizontal feasible bound (*HFB*) and is designed to guide the model toward realistic horizontal alignments. The PI's are searched within the specified gates during the model's optimization process and determine the track of the horizontal alignments. Finally, the alignments resulting from the feasible PI's are obtained as candidates to be evaluated with detailed cost components embedded in the HAO model. The additional allowable offset, D_{offset} is approximately estimated with a simple equation determined by:

$$D_{offset} = R_i \times \left(1/\cos(\alpha_i/2) - 1\right) \tag{4}$$

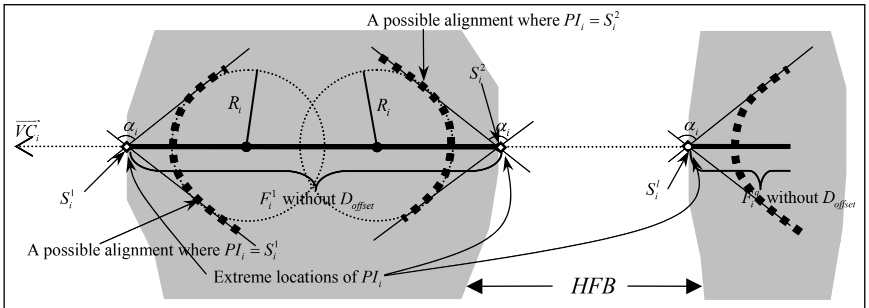
where R_i and α_i are the horizontal curve radius and the deflection angle at PI_i, respectively, as shown in Figure 5. The allowable offset must be added to the horizontal feasible gates to avoid losing good candidate alignments since it is possible that excellent solutions run near borders between the specified feasible bounds and others, as shown in Figure 5(b). Figure 5(a) shows a limit of the horizontal feasible gate approach in a case where no allowable offsets are provided. Some caution is required in determining the deflection angle in order to fully use of the proposed horizontal feasible gate (HFG) approach. The allowable offset (D_{offset}) becomes too long if α_i is too small (e.g., less than 30º) and R_i is too short; thus, we hardly expect the benefit of the proposed HFG approach since the long allowable offset may cover the entire length of the vertical cutting line. It is noted that the minimum curve radius (lower bound of R_i) is determined by design speed, maximum superelevation, and side friction factor. We summarize the feasible gate determination procedure as follows:

Feasible Gate Determination Procedure

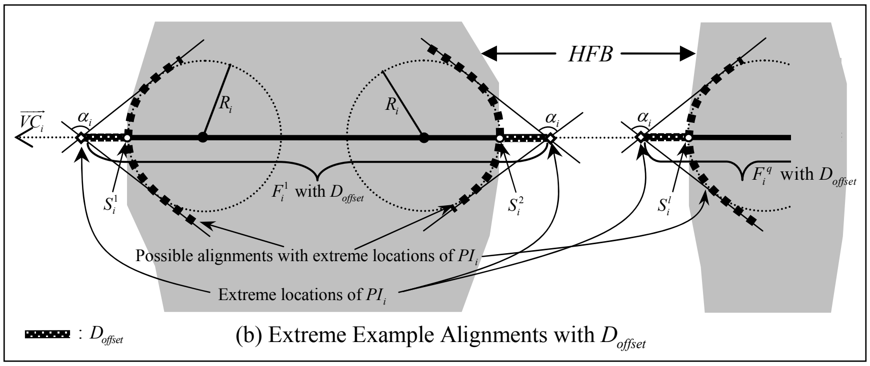
```
STEP 1: for $i = 1$ to $n$
            find $S_i^l$ and $m_i \leftarrow$ only if $\overrightarrow{VC_i}$ intersects $k$ in $HFB$ $\forall k$ in $HFB$
        end

STEP 2: for $l = 1$ to $m_i$
            discard duplicate $S_i^l$
        end
        update $m_i$

STEP 3: for $i = 1$ to $n$
            for $q = 1$ to $m_i / 2$
                for $l = 1$ to $m_i$
                    $F_i^q = \overline{S_i^l \ S_i^{l+1}}$
                    add $D_{offset}$ at the both ends of $F_i^q$
                end
            end
        end
```

3.1.3. User-Defined Constraint for Guiding Horizontal Feasible Alignments

We have set the horizontal feasible bound (*HFB*) and represented the feasible gates of PI's for horizontal alignments realistically through section 3.1.1 and 3.1.2. It is noted however that the derived feasible gates does not always guarantee that feasible alignments are generated that satisfy the user's preferences (i.e., specific constraints). For instance, solution alignments generated from the optimization model might still affect the untouchable areas (*HFB'*) if they are surrounded by or in the middle of the feasible bounds (*HFB)* as shown in Figure 6(a). In addition, the alignments might affect areas of property piece *k* more than allowable amounts specified by users, as shown in Figure 6(b).



(a) Extreme Example Alignments without D_{offset}



(b) Extreme Example Alignments with D_{offset}

Figure 5. Representation of Allowable Offset Near the Feasible Gates

We let A_k and $MaxA_k$ be the area of property piece k and its maximum allowable area affected by the alignment, respectively. $MaxA_k$ is initially set to A_k for the property piece k inside the *HFB* and 0 for outside of it; $MaxA_k$ can be interactively modified by users with the developed GIS module (IDPM). Then a typical constraint to deal with such problems can be expressed as:

$$P_k = \begin{cases} \beta_0 + \beta_1 \times (MaxA_k - A_k^j)^{\beta_2}, & \text{If } MaxA_k < A_k^j \\ 0, & , \ Otherwise \end{cases} \tag{5}$$

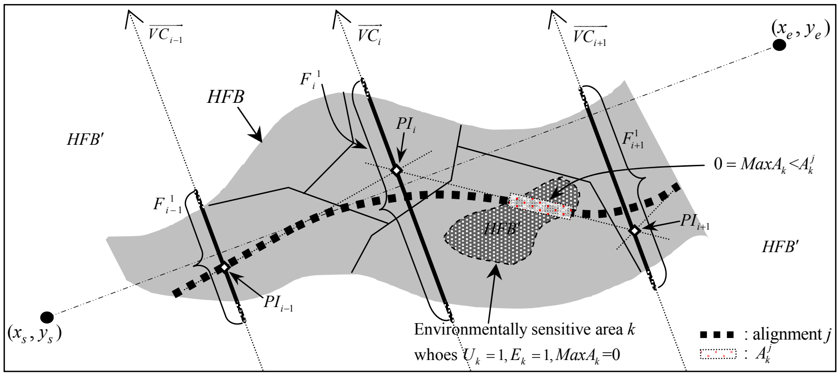
where A_k^j = Affected area of property piece k by alignment j

P_k = Penalty associated with area of property piece k affected by alignment j

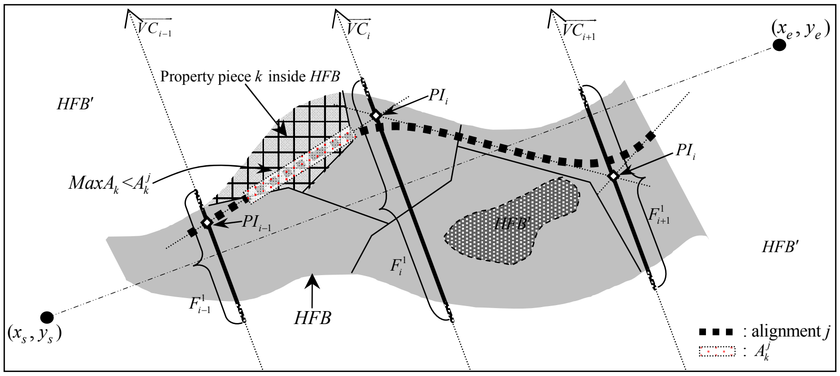
β_0, β_1, and β_2 are user-defined parameters.

The function, $P_{k,}$ called the soft penalty, is widely used in many studies (Jha, 2000; Jha and Schonfeld, 2004) and is intended to smoothly guide the search in the optimization model. A penalty is assigned to the objective function value of the alignment if it violates the user-defined constraint (5).

Table 2 presents attributes of the baseline study area map created from IDPM. Rows shaded in the table represent property pieces in the defined horizontal feasible bound (*HFB*). As stated previously, each property piece k has index variables, U_k and E_k identifying if it is inside U and E, respectively. There are unit property cost and area of k (denoted by C_k and A_k, respectively) in the table to calculate the right-of-way cost of the alignment. In addition, $MaxA_k$ and *Land-use* are also included in the attribute table to reflect the user-defined constraint and estimate environmental impacts of the alignment, respectively.



(a) An Example Alignment Affeting the environmentally Sensitive Area



(b) An Example Alignment Affeting User-Specified Property Piece k with $MaxA_k < A_k^j$

Figure 6. Example Alignments Violating the User-Defined Constraint

The proposed horizontal feasible gate (HFG) method can also be applied to the fixed points in which a new alignment intersects with an existing road and stream or user-specified points. Each of those may require different specific constraints. For instance, constraints might limit the number of intersections if an alignment should not intersect an existing highway more than twice. Constraints might also limit the minimum vertical clearance if the alignment should pass over the existing highway. The proposed approach is applicable to many other cases if corresponding GIS data are available. The horizontal design constraints, such as the minimum curve radius with a given design speed and minimum superelevation runoff length are also embedded in the model but not repeated in this paper since these are outside its scope.

Table 2. Attributes of the Baseline Study Area (*BSA*) Map Created from IDPM

Shape	*k*	U_k	E_k	C_k	A_k	$MaxA_k$	*Land use*
Polygon	1	1	0	0.15	3,504	3,504	Farm
Polygon	2	0	1	0.01	1,000	0	Wetland
Polygon	3	1	0	10.20	2,035	200	Resident
Polygon	4	1	0	11.04	890	100	Resident
Polygon	5	0	0	0.25	4,082	0	Park
Polygon	6	0	0	13.44	2,150	0	Commercial
Polygon	7	0	0	12.63	1,830	0	Resident
Polygon	8	1	0	0.02	1,632	1,632	Stream
Polygon	9	0	1	2.16	1,024	0	Historic
Polygon	10	1	0	0.88	851	100	Historic
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

3.2. Vertical Feasible Gates (VFG) for Optimal Search

To represent the vertical feasible gates (VFG) of an alignment, we adopt the orthogonal cutting plane method developed in Jong (1998) and Jong and Schonfeld (2003), which is an extension of the vertical cutting line concept to the vertical alignment optimization. We first let the *HZ* plane be a coordinate system designed to represent ground and road elevation along the horizontal alignment. The *H* and *Z* axes represent road distance and elevation along the horizontal alignment, respectively. We now define a vertical alignment on the *HZ* plane. Let $Start_V = (H_0, Z_0)$ and $End_V = (H_{n+1}, Z_{n+1})$ be start and end points of the vertical alignment, respectively where $H_0=0$ and we assume that Z_0, H_{n+1}, and Z_{n+1} are given. Then the set of vertical points of intersection can be denoted by $VPI_i = (H_i, Z_i)$ $\forall i=1, \ldots, n$ as shown in Figure 8. The set of consecutive points generally outlines the track of the alignment, while linking each pair of successive points with a straight line produces a piecewise linear trajectory of the alignment (Jong, 1998). The set of vertical feasible gates for *VPI's*, denoted by V_i $\forall i=1, \ldots, n$ are placed in the orthogonal cutting planes (denoted by OC_i $\forall i=1, \ldots, n$) and bounded by upper and lower bounds, Z_i^{LB} and Z_i^{UB} $\forall i=1, \ldots, n$, respectively. Those bounds are determined with the elevations at the previous and subsequent intersection points and a pre-specified maximum gradient, *Gmax.* We now summarize the elevation determination procedure as follows:

Road Elevation Determination Procedure

Find Z_i (for $i=1, \ldots, n$) given with Z_0, Z_{n+1}, *Gmax*, and H_i

STEP 1: Calculate $tempL_i^1$ and $tempU_i^1$

$$tempL_i^1 = Z_{i-1} - (H_i - H_{i-1}) \times Gmax/100$$
$$tempU_i^1 = Z_{i-1} + (H_i - H_{i-1}) \times Gmax/100$$

STEP 2: Calculate $tempL_i^2$ and $tempU_i^2$

$$tempL_i^2 = Z_{n+1} - (H_{n+1} - H_{i-1}) \times Gmax/100$$
$$tempU_i^2 = Z_{n+1} + (H_{n+1} - H_{i-1}) \times Gmax/100$$

STEP 3: Calculate Z_i^{LB} and Z_i^{UB}

$$Z_i^{LB} = \mathrm{Max}(tempL_i^1,\ tempL_i^2)$$
$$= \text{Maximum between } tempL_i^1 \text{ and } tempL_i^2$$
$$Z_i^{UB} = \mathrm{Min}(tempU_i^1,\ tempU_i^2)$$
$$= \text{Minimum between } tempU_i^1 \text{ and } tempU_i^2$$

STEP 4-1: Find Z_i as close as Z_i^g

Case 1: if $Z_i^g < Z_i^{LB}$

$\rightarrow Z_i = Z_i^{LB}$

Case 2: if $Z_i^{LB} \leq Z_i^g \leq Z_i^{UB}$

$\rightarrow Z_i = Z_i^g$

Case 3: if $Z_i^g > Z_i^{UB}$

$\rightarrow Z_i = Z_i^{UB}$

STEP 4-2: Find Z_i randomly between Z_i^{LB} and Z_i^{UB}

$\rightarrow Z_i = r_c[Z_i^{LB},\ Z_i^{UB}]$

where,

H_i = H coordinate of VPI_i, $\forall i = 1, \ldots, n$

Z_i = Z coordinate at VPI_i $\forall i = 1, \ldots, n$

Z_i^g = Ground elevation at H_i

$Gmax$ = Maximum gradient(%) defined by users

$Start_V$ = Start point of a vertical alignment, which is given; $Start_V = (H_0, Z_0)$ where $H_0 = 0$ and Z_0 is given

End_V = Endpoint of a vertical alignment, which is given; $End_V = (H_{n+1}, Z_{n+1})$ where H_{n+1} = alignment length and Z_{n+1} is given

$tempL_i^1$ = Provisional lower bound of Z_i based on Z_{i-1}

$tempL_i^2$ = Provisional lower bound of Z_i based on Z_{n+1}

$tempU_i^1$ = Provisional upper bound of Z_i based on Z_{i-1}

$tempU_i^2$ = Provisional upper bound of Z_i based on Z_{n+1}

Z_i^{LB} = Lower bound of Z_i

Z_i^{UB} = Upper bound of Z_i

$r_c[A, B]$ = A random value from a continuous uniform distribution whose domain is within the interval $[A, B]$

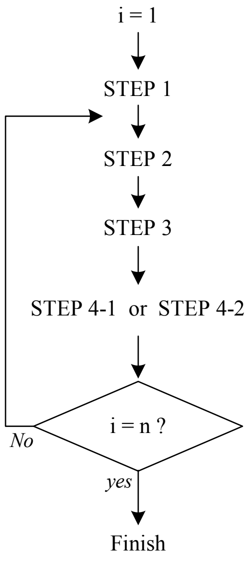


Figure 7. The Elevation Determination Procedure

The ways of dealing with other vertical design constraints (such as minimum length of crest and sag vertical curves) are presented in earlier publications (Jong, 1998; Jong and Schonfeld 2003).

IV. Example Study

Two example scenarios are tested for the Brookeville Bypass case (Kang et al., 2005) to demonstrate the performance of the proposed method. One is the solution search with the original search bound in the previous HAO model and the other is that with the feasible gate (FG) approach. The baseline major design standards used in this example study are a two-lane road with a 40 foot (12.2 meter) cross-section (12 feet (3.7 meters) for lanes and 8 feet (2.4 meters) for shoulders), a 50 mph (80.5 kph) design speed, 5% maximum allowable gradient and 6% maximum superelevation. The model runs 300 generations for each case on a Pentium 4 CPU 3.2GHz with 2GB RAM. The user-specifiable deflection angle, α_i for calculating the allowable offset, D_{offset} is set at 90º in this example.

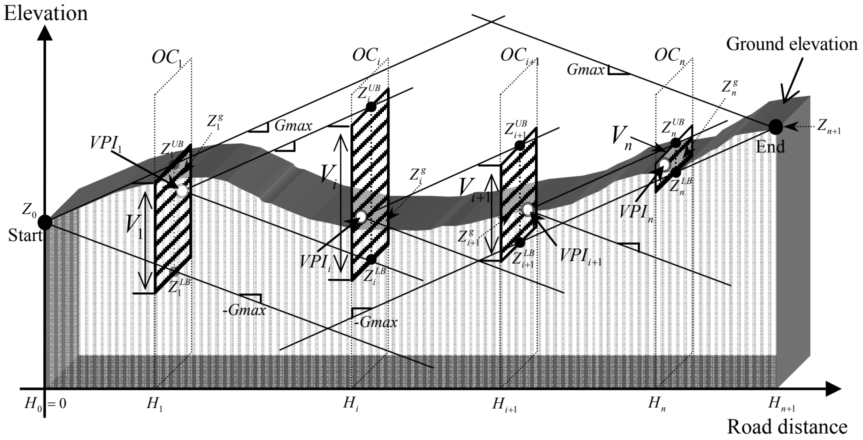


Figure 8. Representation of Feasible Gates for VPI's on *HZ* plane

To incorporate the horizontal feasible gate (HFG) method in the model, Maryland Property View is used as the baseline map and various land use layers (such as wetlands, historic districts, residential areas, and farms) and a horizontal feasible-boundary map defined by users are superimposed on the map. As shown in Figure 9, five horizontal feasible gates for PI's realistically represent the user-defined boundary.

The allowable offset, which is calculated based on 90º deflection angle and the minimum curve radius for the 50 mph design speed, is added to every feasible gate. Example solution alignments generated with the HFG method are successfully placed within the defined horizontal feasible bound.

To test how fast each method finds a reasonable solution, we set a solution boundary based on the optimized solution obtained within 1,000 generations for the same example problem. A "reasonable solution" is defined to be within 2% of the best known solution. Table 3 shows that the model tested with the original method finds a reasonable solution in 5,311 seconds (88.52 minutes). However, with the proposed FG method the model finds such a solution in 3,831 seconds (63.85 minutes), with 27.87% savings in computation time. It is noted here that such a computation time saving can significantly be improved if the scale of the road project is enlarged (e.g., the airline distance between endpoints is longer and geographic entities comprised in the study area increase). In the presented Brookeville example case, the size of the horizontal study area and the airline distance between two endpoints shown in Figure 9 are 3600 feet × 8400 feet (1097 meters × 2560 meters) and 4,003 feet (1,220 meters), respectively. The study area comprises about 650 geographic entities (land properties, structures, roads, etc.).

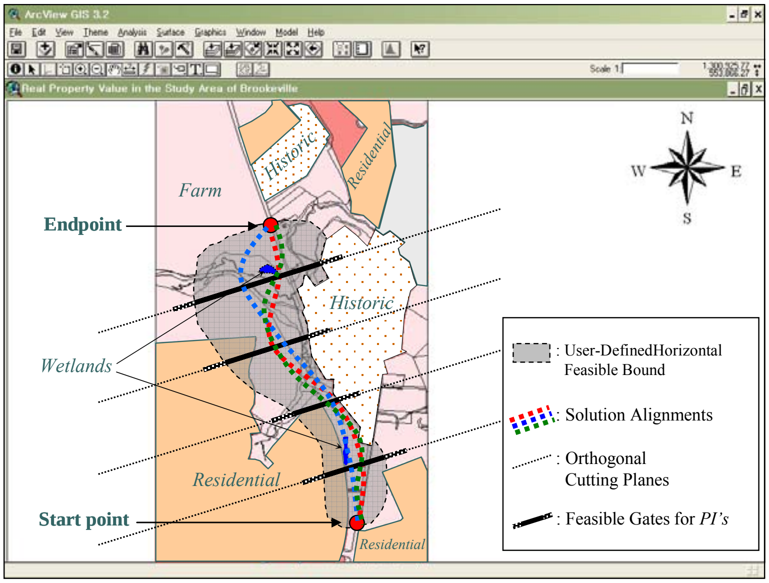


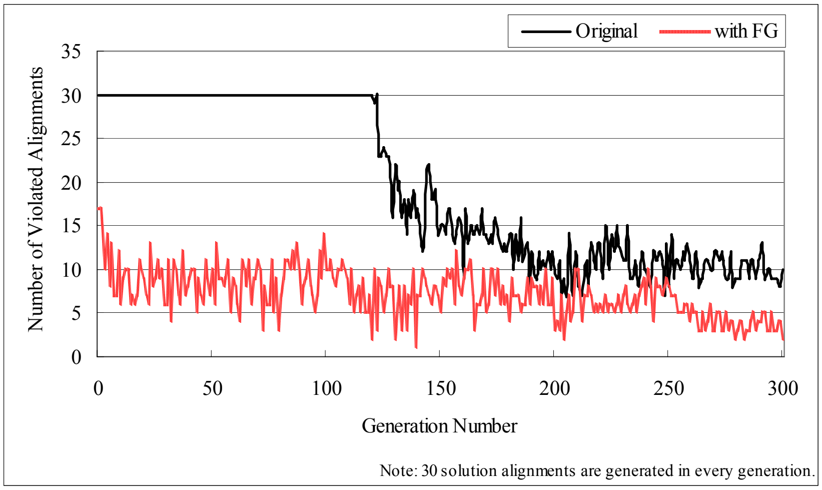
Figure 9. Example Solution Alignments with the FG Method for the Brookeville Project

Table 3. Computation Time Comparison with & without FG Method for the Brookeville Project

Case	Original	With FG
The solution which first enters the 2% bound of the best solutions($)*	4,387,534	4,387,209
(% of the best solution)	(102.00 %)	(102.00 %)
Program computation time to reach the 2% bound of the best solution (sec)	5,311	3,831
Computation time (%)	100.00 %	72.13 %

* The optimized solution obtained with 1,000 generations (Total cost = $4,301,307) is assumed to be the best.
Note: The model runs 300 generations on Pentium 4 CPU 3.2GHz with 2GB RAM and considers supplier costs only.

Figure 10 and Table 4 show how the solution quality improves over successive generations. With the proposed FG method, the numbers of solution alignments violating the specified constraints, which include the user-defined horizontal and vertical bound constraints (i.e., horizontal and vertical feasible gates and maximum allowable areas), significantly decrease in early generations as shown in Figure 10. About 25% of the solutions with the FG method violate those constraints; however, most solutions with the original method have the constraint violations in early generations. Such an effect can also be found in Table 4 showing that total cost breakdowns for the solution alignments at intermediate generations. The solution improvements (i.e., total cost improvements including various cost components) with the proposed method level off earlier than with the original method; the reasonable solution (defined to be within 2% of the best known solution) is found between 150 and 200 generations with the FG method, rather than 250 to 300 generations with the original method. This can be interpreted to indicate that the search process in the model avoids the severely infeasible solutions much sooner and concentrates on refining good solutions with the FG method. With the FG method Penalty1, which indicates a penalty cost for violating the bound constraints that guide horizontally feasible alignments, slightly affects the total costs of the solution alignments in early generations since the solutions slightly exceed the specified allowable limit of areas; however, the penalty disappears soon in later generations. In addition, Penalty 2, which indicates a penalty cost for violating the bound constraints that guide vertically feasible alignments, does not influence the total cost (Penalty2=0) since the FG method guides the model to avoid producing vertical alignments outside the feasible gates.



Note: 30 solution alignments are generated in every generation.

Figure 10. Number of Solution Alignments Violating the User-Defined Horizontal and Vertical Bound Constraints over Successive Generations

Table 4. Solution Quality Comparison with & without the FG Method for the Brookeville Project

Original		Total cost breakdown ($)								Road length (ft)
Gneration No.	Total cost ($)	Construction & Pavement	Right-Of-Way	Earthwork	Bridge	Structure	Penalty1*	Penalty2**	Penalty3***	
25	1,171,320,706	1,772,371	23,145,300	32,605,080	958,640	30,201	1,043,540,000	69,034,532	234,582	4168
50	344,967,932	1,955,606	55,259	4,879,417	892,925	109,864	235,870,600	101,087,400	116,861	4627
100	342,174,338	1,881,829	52,208	2,207,828	915,010	88,891	235,868,500	101,086,500	73,572	4442
150	6,563,385	1,841,228	49,648	3,573,298	916,665	89,449	48,903	12,048	32,146	4341
200	4,602,704	1,840,343	49,764	1,682,728	922,045	76,993	14,040	7,539	9,252	4338
250	4,407,148	1,834,924	49,851	1,530,583	911,240	76,993	3,557	0	0	4335
300	4,358,840	1,839,910	49,747	1,495,842	906,021	67,320	0	0	0	4337

with FG		Total cost breakdown ($)								Road length (ft)
Gneration No.	Total cost ($)	Construction & Pavement	Right-Of-Way	Earthwork	Bridge	Structure	Penalty1*	Penalty2**	Penalty3***	
25	6,039,951	1,897,966	52,309	2,906,272	925,805	83,778	10,000	0	163,821	4482
50	4,978,707	1,883,711	51,797	1,910,133	896,290	78,589	11,873	0	146,314	4447
100	4,634,584	1,825,958	49,442	1,767,422	894,305	55,358	20,382	0	21,717	4302
150	4,390,198	1,825,644	49,462	1,507,529	894,470	64,157	5,648	0	43,289	4302
200	4,365,783	1,845,601	49,816	1,495,630	897,162	67,253	0	0	10,321	4312
250	4,328,416	1,825,704	49,459	1,494,586	894,510	64,157	0	0	0	4302
300	4,328,416	1,825,704	49,459	1,494,586	894,510	64,157	0	0	0	4302

* Penalty cost for violating the specified horizontal feasible bound constraints (i.e., untouchable areas and maximum allowable areas).

** Penalty cost for violating the specified vertical feasible bound constraints (i.e., ranges of vertical feasible gates).

*** Penalty cost for violating other design constraints (e.g., minimum horizontal curve radius and minimum length of vertical curve).

Note: The model runs 300 generations on Pentium 4 CPU 3.2GHz with 2GB RAM and considers supplier costs only.

V. Conclusions and Future Work

5.1. Conclusions

An efficient optimization method called feasible gate (FG) (for horizontal (HFG) and vertical (VFG) alignments) is developed to improve the computation efficiency and solution quality of the previously developed highway alignment optimization (HAO) model. It improves the search efficiency of the model by maximizing the chance that alignments satisfying certain environmental, user-preference and geometric constraints are generated. This is achieved by generating points of intersection (PI's) for alignments only within some appropriately limited subsets ("gates") of the orthogonal cutting planes. A customized GIS module (IDPM) is also developed for integrating the proposed method and the HAO model.

Two test examples with a real road project show how the proposed method improves the model's solution quality and reduces its computation time. Through a realistic application of the model with the FG method, it has been found that the model's computation time is reduced by approximately 28% reduction, as shown in Table 3, and its solution quality is improved throughout the search process, as shown in Figure 10 and Table 4. It is noted that the improvement due to the FG method can significantly increase if the scale of the road project is enlarged (e.g., in the number of geographic entities in the study area)

The HAO model can now represent a complex road project realistically and evaluate numerous alignments that satisfy various user preferences since the FG method assists the model in narrowing its horizontal and vertical feasible bounds based on the specified conditions including user preferences. Thus, it can focus sooner on refining the feasible alignments and provides the optimized solutions much faster. The authors expect the proposed FG approach to be especially applicable in improving existing roads, such as by widening them within very limited bounds, besides optimizing completely new alignments.

Some caution is required in using the FG method. The effect of the FG method would be negligible if the allowable offset (D_{offset}) added to the horizontal gates is excessive; i.e., the horizontal feasible gates for PI's might cover the entire search spaces of original method if the offset is excessive. On the other hand, it is possible to lose good candidate alignments if the offset is too short (excellent solutions may run near borders between the specified feasible bounds and others, as shown in Figure 5(b)).

5.2. Future Extensions

Even with the proposed FG method, the HAO model still can be improved in various ways. The following are some issues to be considered in the near future for enhancing the model's performance.

Optimized Endpoints

The fixed endpoints-assumption can be relaxed to undefined endpoints by incorporating in the model a detailed cost or decision model for crossing structures along the existing road. Such a relaxation is desirable since the endpoints of new alignments are often not prespecified in early stages of road planning.

Prescreening and Repairing Process

In order to reduce computation time, a prescreening process should be added. This process should be used to quickly eliminate undesirable solutions (such as alignments that do not satisfy AASHTO's design standards) early in the search process, before detailed evaluation. A multi-stage screening process based on relative computation time of the various cost components might also be desirable for the model's computation efficiency.

The Number of PI's

The number of PI's is a key input parameter in the precision of the solutions since it affects location of horizontal and vertical curve sections as well as corresponding cost-components embedded in the model. In dense urban areas and areas with significant variation in the topography, a higher PI density will improve the possibilities for optimization, whereas in areas with slight variation in topography or land-use, fewer PI's will suffice. Therefore, the number of PI's should be related to the complexity of the search space.

Optimizing Alignments for Simple Networks

Optimization models for simple highway networks are being explored as the next major extension of the HAO model.

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NOTATION

The following symbols are used in this article:

A_k = Area of property piece k (e.g., 10,000 sq.ft.)

A_k^j = Area of property piece k affected by alignment j

D_s = Distance between consecutive vertical cutting lines

BSA = A baseline study area with property information: rectangular space

C_k = Unit cost of property piece k (e.g., \$10/ sq.ft.)

D_{offset} = Allowable additional offset for feasible gates

E = Environmentally sensitive areas or critical control areas

E' = Areas outside E or envrionmentally insensitive areas

E_k = Index variable identifying if property piece k is inside E

End_H = Endpoint of a horizontal alignment, which is given; $End_H = (x_e, y_e)$

End_V = Endpoint of a vertical alignment, which is given; $End_V = (H_{n+1}, Z_{n+1})$
where H_{n+1} = alignment length and Z_{n+1} is given

F_i^q = q_{th} horizontal feasible gates for PI_i (for $i = 1, \ldots, n$; $q = 1, \ldots, m_i/2$)
where $m_i/2$ = number of feasible gates for PI_i

$Gmax$ = Maximum gradient(%) defined by users

h = Height of the baseline study area (BSA)

H_i = H coordinate of VPI_i (for $i = 1, \ldots, n$)

HFB = Horizontal feasible bounds

HFB' = Outside HFB or untouchable areas

k = A clipped property piece in the baseline study area(BSA)

$MaxA_k$ = Maximum allowable areas of property piece k
affected by an alignment (e.g., 500 sq.ft.)

m_i = The number of intersection point of $\overrightarrow{VC_i}$ with k in HFB

n = Number of the vertical cutting lines

O_i = Origin of the i_{th} vertical cutting line, (for $i = 1, \ldots, n$); $O_i = (x_{o_i}, y_{o_i})$

OB = Origin of the baseline study area(BSA); $OB = (x_{origin}, y_{origin})$

OC_i = i_{th} Orthogonal cutting plane, (for $i = 1, \ldots, n$)

P_k = Penalty associated with area of property piece k affected by alignment j

PI_i = i_{th} horizontal point of intersection (for $i = 1, \ldots, n$); $PI_i = (x_i, y_i)$

$r_c[A, B]$ = A random value from a continuous uniform distribution
whose domain is within the interval $[A, B]$

R_i = Designed horizontal curve radius at PI_i

S_i^l = l_{th} intersection point of $\overrightarrow{VC_i}$ with k in HFB (for $i = 1, \ldots, n$; $l = 1, \ldots, m_i$);
= $(x_{S_i^l}, y_{S_i^l})$

$Start_H$ = Start point of a horizontal alignment, which is given ; $Start_H = (x_s, y_s)$

$Start_V$ = Start point of a vertical alignment; $Start_V = (H_0, Z_0)$
where $H_0 = 0$ and Z_0 is given

$tempL_i^1$ = Provisional lower bound of Z_i based on Z_{i-1}

$tempL_i^2$ = Provisional lower bound of Z_i based on Z_{n+1}

$tempU_i^1$ = Provisional upper bound of Z_i based on Z_{i-1}

$tempU_i^2$ = Provisional upper bound of Z_i based on Z_{n+1}

U = Areas of interest or prefered areas

U' = Areas outside U

U_k = Index variable identifying if property piece k is inside U

V_i = Feasible gate of VPI_i bounded by Z_i^{LB} and Z_i^{UB}

VC_i = i_{th} vertical cutting line, (for $i = 1, \ldots, n$)

$\overrightarrow{VC_i}$ = i_{th} vertical cutting line vector, (for $i = 1, \ldots, n$)

VPI_i = i_{th} vertical point of intersection on HZ plane (for $i = 1, \ldots, n$);
= (H_i, Z_i)

w = Width of the baseline study area (BSA)

Z_i = Z coordinate at VPI_i (for $i = 1, \ldots, n$)

Z_i^g = Ground elevation at H_i

Z_i^{LB} = Lower bound of Z_i computed with $tempL_i^1$ and $tempL_i^2$

Z_i^{UB} = Upper bound of Z_i computed with $tempU_i^1$ and $tempU_i^2$

α_i = Deflection angle at PI_i

θ = The angle between the vertical cutting line and the X axis